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MONDAY, JUNE 9TH, 1856.

JAMES HENTHORN TODD, D. D., PRESIDENT,
in the Chair.

DOMINICK M'CAUSLAND, Robert M'Dermott, M. B., Rev. James M'Ivor, and Sir Colman M. O'Loghlen, Bart., were elected Members of the Academy.

Rev. Charles Graves, D. D., read a letter from Dr. Boole, on the solution of the equation of continuity of an incompressible fluid.

"*Queen's College, Cork,*
"*5th May,* 1856.

"MY DEAR GRAVES,—Your memoirs on the application of the method of Quaternions to the solution of the partial differential equation,

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0,$$

have called to my mind some researches upon which I was engaged many years ago, and which establish what I conceive to be the general theory of such applications, and throw some light upon the question of the probable value of the results obtained. I will communicate to you the fundamental theorem to which my investigations led, and explain its application to the particular case which you consider. But I wish you to understand that this application was not contemplated by me at the time the theorem was discovered.

"First, however, I will observe that your method expressly consists in interpreting the forms

$$e^{-x(jD_2+kD_3)}f_1(y, z), \quad e^{x(jD_2+kD_3)}f_2(y, z),$$

where D_2 stands for $\frac{d}{dy}$, and D_3 for $\frac{d}{dz}$,—forms originally suggested by Mr. Carmichael, but by him erroneously interpreted. The compound symbols which operate upon the arbitrary functions $f_1(y, z)$, $f_2(y, z)$, are both functions of the symbol $jxD_2 + kxD_3$, which is, in fact, a special form of a quaternion. It is what the quaternion $p + qi + rj + sk$ becomes if we make $p = 0$, $q = 0$, $r = xD_2$, $s = xD_3$, assumptions which do not violate any of the formal conditions to which quaternions are subject. For the symbols xD_2, xD_3 combine with each other like constant *quantities.*

"It appears to me, therefore, that the operating symbols being functions of symbolical quaternions, the most natural method of seeking their interpretation is to refer them to the more general problem of the development of a function of a quaternion. This development constitutes the object of the fundamental theorem to which I have referred. It is contained in the following

PROPOSITION.

"To develop $f(w + ix + jy + kz)$ in the form of a simple quaternion, i. e. in the form $W + iX + jY + kZ$, where W, X, Y, Z are functions of w, x, y, z; it being given that $f(w)$ is capable of being developed in ascending powers of w by Maclaurin's theorem.

"Since $f(w)$ is capable of being expressed in a series of the form $A + Bw + Cw^2 + Dw^3 + \&c.$, it is evident that if we represent the quaternion $w + ix + jy + kz$ by Q, the function $f(Q)$ will be capable of expansion in the corresponding series $A + BQ + CQ^2 + DQ^3 + \&c.$ (1), for Q combines with symbols of quantity just as if it were itself a symbol of quantity. We might indeed consider $f(Q)$ as intelligible only by means of its development in a series of integral powers of Q. Thus we might take as the very *definition* of e^Q that it represents the series $1 + Q + \frac{1}{1.2}Q^2 + \frac{1}{1.2.3}Q^3 + \&c.$ It is only by reference to such a series that we can see how a function of a quaternion such as e^Q can be reduced to a simple quaternion. For the several powers Q^2, Q^3, &c. assume by actual involution the forms of simple quaternions, and their substitution in the series reduces it to a quaternion also, the coefficients W, X, Y, Z being expressed by infinite series.

"From the form of $f(Q)$, as expressed in (1), it is evident that we shall have $Qf(Q) = f(Q)\,Q$. Now substituting for Q and $f(Q)$ their values, viz.

$$\left.\begin{aligned} Q &= w + ix + jy + kz \\ f(Q) &= W + iX + jY + kZ \end{aligned}\right\} \qquad (2)$$

we have

$$\begin{aligned}(w + ix + jy + kz)\,(W + iX + jY + kZ)& \\ = (W + iX + jY + kZ)\,(w + ix + jy + kz)&. \qquad (3)\end{aligned}$$

an equation which, it is to be observed, would not be true unless the quaternions $w + ix + jy + kz$ and $W + iX + jY + kz$ were functionally related. Multiplying out, and attending to the rules of quaternions, we have from (3)

$$\begin{aligned} w^2 - xX - yY - zZ + i(wX + Wx + yZ - Yz) + j(wY + Wy \\ + zX - Zx) + k(wZ + Wz + xY - Xy) = w^2 - Xx - Yy \\ - Zz + i(Wx + Xw + Yz - yZ) + j(Wy + wY + Zx - zX) \\ + k(Wz + wZ + Xy - xY). \end{aligned}$$

Equating coefficients, we find

$$yZ - Yz = 0, \quad zX - Zx = 0, \quad xY - Xy = 0,$$

or

$$\frac{X}{x} = \frac{Y}{y} = \frac{Z}{z} = V, \text{ suppose.}$$

We are therefore permitted to assume

$$f(w + ix + jy + kz) = W + V(ix + jy + kz) \qquad (4)$$

and it remains to determine W and V.

"Now $f(Q)$ being by hypothesis of the form $\Sigma A_n Q^n$, let us seek the special forms of W and V for the particular case in which $f(Q) = Q^n$.

"In virtue of (4) we may then write

$$Q^n = W_n + V_n(ix + jy + kz), \qquad (5)$$

whence

$$Q^{n+1} = W_{n+1} + V_{n+1}(ix + jy + kz). \qquad (6)$$

Now multiplying both sides of (5) by Q, or $w + ix + jy + kz$, we have

$$Q^{n+1} = wW_n - V_n(x^2 + y^2 + z^2) + (wV_n + W_n)(ix + jy + kz).$$

Comparing this with (6), we have

$$\left.\begin{aligned} W_{n+1} &= wW_n - r^2V_n \\ V_{n+1} &= wV_n \;+\; W_n \end{aligned}\right\} \text{ where } r^2 = x^2 + y^2 + z^2,$$

a pair of simultaneous equations of finite differences. From these we readily deduce

$$W_{n+2} - 2wW_{n+1} + (w^2 + r^2)\,W_n = 0,$$
$$V_{n+2} - 2wV_{n+1} + (w^2 + r^2)\,V_n = 0.$$

The complete integrals of which are

$$W_n = c(w + r\sqrt{-1})^n + c'(w - r\sqrt{-1})^n, \qquad (7)$$
$$V_n = b(w + r\sqrt{-1})^n + b'(w - r\sqrt{-1})^n. \qquad (8)$$

c, c, b, b', being arbitrary constants.

"To determine the values of these constants let $n = 0$; whence from (5) it is evident that $W_0 = w$, $V_0 = 0$; values which, substituted in (7) and (8), give

$$1 = c + c',$$
$$0 = b + b'.$$

"Again, let $n = 1$, whence substituting in (5) for Q its value $w + ix + jy + kz$, we find $W_1 = w$, $V_1 = 1$, and employing these values in (7) and (8), we have

$$w = c(w + r\sqrt{-1}) + c'(w - r\sqrt{-1}),$$
$$1 = b(w + r\sqrt{-1}) + b'(w - r\sqrt{-1}).$$

From these four equations, we find

$$c = \frac{1}{2}, \quad c' = -\frac{1}{2}, \quad b = \frac{1}{2r\sqrt{-1}}, \quad b' = \frac{-1}{2r\sqrt{-1}}.$$

Substituting these values in (7) and (8), and the resulting values of W_n and V_n in (5), we have

$$Q^n = \frac{(w + r\sqrt{-1})^n + (w - r\sqrt{-1})^n}{2}$$
$$+ \frac{(w + r\sqrt{-1})^n + (w - r\sqrt{-1})^n}{2r\sqrt{-1}}(ix + jy + kz). \quad (9)$$

Multiply both members of this equation by A_n, and attach the symbol Σ. Then since

$$\begin{aligned}\Sigma A_n Q^n &= f(Q) = f(w + ix + jy + kz)\\ \Sigma A_n (w + r\sqrt{-1})^n &= f(w + r\sqrt{-1})\\ \Sigma A_n (w - r\sqrt{-1})^n &= f(w - r\sqrt{-1})\end{aligned}$$

we have

$$f(w + ix + jy + kz) = \frac{f(w + r\sqrt{-1}) + f(w - r\sqrt{-1})}{2}$$
$$+\frac{f(w + r\sqrt{-1}) - f(w - r\sqrt{-1})}{2r\sqrt{-1}}(ix + jy + kz) \quad \text{(I.)}$$

the development required.

PARTICULAR DEDUCTIONS.

“ There are two particular cases of the above theorem which deserve special notice.

“ The first is when $f(Q) = Q^n$. The expression given in (9) may then be reduced to the following form:

$$(w + ix + jy + kz)^n = (w^2 + r^2)^{\frac{n}{2}}\left(\cos n\theta + \frac{\sin n\theta}{r}(ix + jy + kz)\right)$$

where $\theta = \tan^{-1}\frac{r}{w}$.

“ If $n = -1$, this gives

$$(w + ix + jy + kz)^{-1} = (w^2 + r^2)^{-1}(w - ix - jy - kz),$$

a well-known theorem.

The second case is when $f(Q) = e^Q$. Here we have

$$e^{w+ix+jy+kz} = \frac{e^{w+r\sqrt{-1}} + e^{w-r\sqrt{-1}}}{2} + \frac{e^{w+r\sqrt{-1}} - e^{w-r\sqrt{-1}}}{2r\sqrt{-1}}(ix + jy + kz)$$
$$= e^w \cos r + e^w \frac{\sin r}{r}(ix + jy + kz)$$

$$\therefore\ e^{w+ix+jy+kz} = e^w\left[\cos r + \frac{\sin r}{r}(ix + jy + kz)\right] \quad \text{(II.)}$$

where, as before, $r = \sqrt{(x^2 + y^2 + z^2)}$.

“ *Application of the above Results to the Solution of the Equation* $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0$:—

“ Writing this equation in the form

$$\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right) u = 0,$$

we can, by the properties of quaternions, reduce it to any of the following forms,

$$\left(\frac{d}{dx} + i\frac{d}{dy} + j\frac{d}{dz}\right)\left(\frac{d}{dx} - i\frac{d}{dy} - j\frac{d}{dz}\right) u = 0,$$

$$\left(\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)\left(\frac{d}{dx} - j\frac{d}{dy} - k\frac{d}{dz}\right) u = 0,$$

$$\left(\frac{d}{dx} + k\frac{d}{dy} + i\frac{d}{dz}\right)\left(\frac{d}{dx} - k\frac{d}{dy} - i\frac{d}{dz}\right) u = 0.$$

You employ the second of these, which leads to the solution

$$u = e^{x\left(j\frac{d}{dy} + k\frac{d}{dz}\right)} f_1(y, z) + e^{-x\left(j\frac{d}{dy} + k\frac{d}{dz}\right)} f_2(y, z),$$

the symbolical form adopted by Mr. Carmichael and yourself.

“ Now the development of the first operating symbol will be obtained from (II.) by changing therein w and x to 0, y to $x\frac{d}{dy}$, and z to $x\frac{d}{dz}$: whence $r = x\sqrt{\left\{\left(\frac{d}{dy}\right)^2 + \left(\frac{d}{dz}\right)^2\right\}}$.

“ We thus find

$$e^{x\left(j\frac{d}{dy} + k\frac{d}{dz}\right)} = \cos\left\{x\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right)^{\frac{1}{2}}\right\} + \frac{\sin\left\{x\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right)^{\frac{1}{2}}\right\}}{x\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right)^{\frac{1}{2}}} \times \left(xj\frac{d}{dy} + xk\frac{d}{dz}\right).$$

The development of $e^{-x\left(j\frac{d}{dy} + k\frac{d}{dz}\right)}$ will be obtained from the above by merely changing x into $-x$.

" If, for simplicity, we represent the expression $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}$ by Δ, we find for u the values

$$u=\cos\left(x\Delta^{\frac{1}{2}}\right)f_1(y,z)+\sin\left(x\Delta^{\frac{1}{2}}\right)\Delta^{-\frac{1}{2}}\left\{\frac{jdf_1(y,z)}{dy}+\frac{kdf_1(y,z)}{dz}\right\}$$
$$+\cos\left(x\Delta^{\frac{1}{2}}\right)f_2(y,z)-\sin\left(x\Delta^{\frac{1}{2}}\right)\Delta^{-\frac{1}{2}}\left\{\frac{jdf_2(y,z)}{dy}+\frac{kdf_2(y,z)}{dz}\right\}$$

and if we assume

$$f_1(y,z)+f_2(y,z)=\phi_1$$
$$f_1(y,z)-f_2(y,z)=\phi_2$$

we find for the general value of u the expression

$$u=\cos\left(x\Delta^{\frac{1}{2}}\right)\phi_1+\sin\left(x\Delta^{\frac{1}{2}}\right)\Delta^{-\frac{1}{2}}\left\{\frac{jd\phi_2}{dy}+\frac{kd\phi_2}{dz}\right\} \qquad \text{(III.)}$$

ϕ_1 and ϕ_2 being arbitrary functions of y and z.

" This solution agrees substantially with the one which you have obtained. If we develop the cosine and the sine, each of the operating functions will assume the form of a series expressed in ascending integral powers of Δ, i. e., of $\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$, and the operation can then be performed when ϕ_1 and ϕ_2 are given. We shall, in fact, have

$$u=F_1+jF_2+kF_3, \text{ where}$$

$$F_1=\phi_1-\frac{x^2}{1.2}\left(\frac{d^2\phi_1}{dy^2}+\frac{d^2\phi_1}{dz^2}\right)+\frac{x^4}{1.2.3.4}\left(\frac{d^4\phi_1}{dx^4}+2\frac{d^4\phi_1}{dx^2dy^2}+\frac{d^4\phi_1}{dy^4}\right)$$
$$-\text{ \&c.}$$

$$F_2=x\frac{d\phi_2}{dy}-\frac{x^3}{1.2.3}\left(\frac{d^3\phi_2}{dy^3}+\frac{d^3\phi_2}{dydz^2}\right)+\text{ \&c.}$$

$$F_3=x\frac{d\phi_2}{dz}-\frac{x^3}{1.2.3}\left(\frac{d^3\phi_2}{dz^3}+\frac{d^3\phi_2}{dy^2dz}\right)+\text{ \&c.}$$

results agreeing with those which you have given in your paper of April 9th, 1855.

" Instead of deducing these results directly from the symbolical forms to which the above analysis leads, your investigations conduct you to a process which consists in substituting in the arbitrary functions ϕ_1, ϕ_2, the expressions $y+jx$, $z+kx$ for y and z respectively, developing the results in ascending powers of x by Taylor's theorem, and then substituting, according to a certain directive canon, 'mean products' of j and k for ordinary products. I think it a remarkable circumstance that it should be possible thus to obtain the true developed values of F_1, F_2, F_3, and the theorem upon which the process is founded is well worthy of being recorded. But I cannot agree with you that it can be considered as virtually freeing your solution, viewed with reference to the determination of F_1, F_2, F_3, from imaginary quantities. For in any parallel case in which imaginary quantities are involved in an algebraic expression, we can, formally at least, get rid of them by substituting for the given expression some other expression not involving those quantities, with the provision that *after* development certain changes, governed by a particular rule or canon, shall be made. The function $\cos x$, for example, considered analytically, involves imaginary quantities, for it is expressed in finite terms by the formula $\frac{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}}{2}$. Now I apprehend that we should not virtually escape from this condition of its finite expression, by presenting the function under the form $\frac{e^x + e^{-x}}{2}$, and adding, as a direction, that in the development of this function the signs of the alternate terms should be changed.

"When among the physical conditions of a problem dependent upon the differential equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0,$$

symmetry, with reference to an axis, as that of x, is involved, a particular integral of the equation may be presented in the form $u = \int_0^\pi d\theta\phi(x + r\cos\theta\sqrt{-1})$, wherein $r = \sqrt{(y^2 + z^2)}$. This integral was given first, so far as I am aware, by myself in the 'Cambridge and Dublin Mathematical Journal,' Jan. 1847, and was shown to represent the most complete solution of the equation, with reference to the problem of the attraction of a solid of revolution on an external point.* See also a memoir by Hoppe, on the problem of the motion of conoidal bodies through an incompressible fluid ('London Quarterly Journal of Mathematical Science,' No. IV.). In its general form, i. e. antecedently to the determination of the function ϕ, this integral cannot be freed from imaginary quantities without development. When, however, the function ϕ is known, it may be freed from them without development. It appears to me, therefore, that the solution possesses, in respect of its relation to imaginary quantities, an advantage over the form which you have deduced by the method of quaternions.

" The direct symbolical solution of the equation,

$$\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}=0,$$

obtained by resolving the operating symbol into factors of the form $\frac{d}{dx}+\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\sqrt{-1}$, and $\frac{d}{dx}-\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\sqrt{-1}$, is

$$u=\cos\left\{x\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\right\}f_1(y,z)+\sin\left\{x\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\right\}f_2(y,z).$$

If, in consequence of the arbitrary form of the functions $f_1(y,z)$, $f_2(y,z)$, we replace the latter function by

$$\left(\frac{d}{dy^2}+\frac{d^2}{dz^2}\right)^{-\frac{1}{2}}\phi_2(y,z),$$

wherein $\phi_2(y,z)$ denotes also an arbitrary function of y and z, and then for symmetry write ϕ_1 for f_1, we have

$$u=\cos\left\{x\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\right\}\phi_1(y,z)+\sin\left\{x\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)^{\frac{1}{2}}\right\}$$
$$\left(\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)\phi_2(y,z),$$

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* I have since obtained, in the form of a double integral, the complete solution of the equation under the condition referred to.

a solution which becomes interpretable by developing the circular functions.

"Now in what respect does this solution differ from the one furnished by the quaternion method, (III.)? Merely in this, that the arbitrary function $\phi_2(y, z)$ in the one, is replaced by the arbitrary quaternion function $\left(j\frac{d}{dy} + k\frac{d}{dz}\right)\phi_2(y, z)$ in the other. Practically, then, the employment of quaternions in the resolution of the factor $\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ merely leads to the substitution of an arbitrary function involving quaternion coefficients for an arbitrary function of the ordinary species. Might we not, then, if there be any advantage in the result, introduce the change at once? If $\phi(y, z)$ be an arbitrary function involved in the solution of a linear equation, it is evident that we may satisfy the equation by replacing that arbitrary function by any other of the form $\Sigma i\phi(y, z)$, i being susceptible of any system of constant values, real or imaginary. It is seen that the quaternion analysis employed from the beginning leads equally to the forms

$$\left(i\frac{d}{dy} + j\frac{d}{dz}\right)\phi_2(y, z) \text{ and } \left(k\frac{d}{dy} + i\frac{d}{dz}\right)\phi_2(y, z),$$

as to the form

$$\left(j\frac{d}{dy} + k\frac{d}{dz}\right)\phi_2(y, z);$$

and you correctly observe that any of the separate terms affected by distinct imaginaries equally satisfy the equation.

"I offer no apology for making these observations. I am sure that your object, like mine, is the discovery of truth alone. The application of quaternions to the solution of partial differential equations is a subject deserving of being thoroughly investigated; partly because of the analytical interest attaching to the inquiry, and partly because the possibility of resolving the function $\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ into two linear factors, seems at first sight to promise material aid in the solution of a problem of peculiar physical importance. The latter consideration appears to have been present to your own mind. I have now stated to you the reasons which have led me to entertain a different opinion.

"Believe me to be, my dear Graves,
"Yours very truly,
"GEORGE BOOLE.

"*The Rev. Dr. Graves.*"

Dr. Graves remarked that the general expression given by Dr. Boole for the development of a function of a quaternion, viz.:—

$$f(w+ix+jy+kz)=\tfrac{1}{2}\{f(w+r\sqrt{-1})+f(w-r\sqrt{-1})\}$$
$$+\frac{1}{2r\sqrt{-1}}\{f(w+r\sqrt{-1})=f(w-r\sqrt{-1})\}(ix+jy+kz),$$

might be obtained by a process simpler, though less interesting, than that adopted by Dr. Boole.

Putting

$$x=r\cos\alpha,\ y=r\cos\beta,\ z=r\cos\gamma,\ \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1;$$

and denoting $i\cos\alpha+j\cos\beta+k\cos\gamma$ by ι; the problem is to develop $f(w+r\iota)$ in the form $W+iX+jY+kZ$.

Now, as w is commutative with r and ι, we may employ Taylor's theorem in the present instance, and thus find

$$f(w+r\iota)=f(w)+f'(w)\,r\iota-\frac{1}{1.2}f''(w)\,r^2-\&c.$$

The symbol ι being a square root of negative unity, this development will be precisely similar to what we should have obtained if we had sought that of $f(w+r\sqrt{-1})$; save only that ι stands in the place of $\sqrt{-1}$. Consequently, we have

$$f(w+r\iota)=\tfrac{1}{2}\{f(w+r\sqrt{-1})+f(w-r\sqrt{-1})\}$$
$$+\frac{\iota}{2\sqrt{-1}}\{f(w+r\sqrt{-1})-f(w-r\sqrt{-1})\},$$

in which, if we put $ix + jy + kz$ in place of r_ι, we obtain the expression given by Dr. Boole.

As regards the functions F_1, F_2, F_3, involved in the expression $F_1 + jF_2 + kF_3$, proposed by Dr. Graves as a complete solution of the equation

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0,$$

(p. 221), he desired it to be understood that he had not intended to claim for them such an independence upon imaginaries as Dr. Boole supposed him to assert. When developed in the form of series, they involve indeed no square roots of negative unity; but analytically they owe their origin to imaginaries of this kind.

Mr. Charles Haliday read a Paper on the Scandinavian Antiquities of Dublin.

The following letter from Mr. Francis M. Jennings was read:—

"*Gibraltar*, *May* 22, 1856.

"My dear Sir,—I have made up, ready to send either by post or private hand, an earring of silver used by the Soos men, at a place about 60 miles south of the city of Morocco. Also, a brooch of silver, such as are used by the Bedouins of Morocco at present; you will see it has been worn. The first I got out of the man's ear; the other from a trader. I keep duplicates, for fear of their being lost; they are for the Academy, if they think them worth accepting. I have got a few other things also, including a necklace; but I think the brooch is the most curious, as its resemblance is so great to the Irish ones. I also got off the *right* arm, from above the elbow, (where it is worn) what would be called a piece of ring-money